

## Curricular Structure for Second Year of Two-year PG Programme under Structure-1 (Only Course work)

| Semester     | DSC                            | DSE  | 2 Credit Course  | Dissertation/<br>Academic Project/<br>Entrepreneurship | Total Credits |
|--------------|--------------------------------|--|--|--|---------------|
| Semester-III | DSC-7<br>DSC-8<br>(8 Credits)  | DSE-5<br>DSE-6<br>DSE-7<br>OR<br>DSE-3, DSE-4 &<br>GE-3<br>(12 Credits)  | Skill-based course/<br>Workshop/ Specialized<br>laboratory/ Internship/<br>Apprenticeship/ Hands<br>on learning<br>(2 Credits) | NIL  | 22            |
| Semester-IV  | DSC-9<br>DSC-10<br>(8 Credits) | DSE-8<br>DSE-9<br>DSE-10<br>OR<br>DSE-5, DSE-6 &<br>GE-4<br>(12 Credits) | Skill-based course/<br>Workshop/ Specialized<br>laboratory/ Internship/<br>Apprenticeship/ Hands<br>on learning<br>(2 Credits) | NIL  | 22            |

## Details of Courses in Second Year of Two-year M.Sc. Mathematics

| Semester     | DSC   | DSE   | SBC                           | GE  |
|--------------|---|---|-------------------------------|---|
| Semester-III | DSC-7: Fluid Dynamics<br>DSC-8: Measure and Integration | <p><b>DSE-5:</b></p> <ul style="list-style-type: none"> <li>(i) Algebraic Topology</li> <li>(ii) Commutative Algebra</li> <li>(iii) Dynamical Systems</li> <li>(iv) Theory of Bounded Operators</li> </ul> <p><b>DSE-6:</b></p> <ul style="list-style-type: none"> <li>(i) Advanced Complex Analysis</li> <li>(ii) Numerical Methods for Ordinary Differential Equations</li> <li>(iii) Representation of Finite Groups</li> <li>(iv) Topological Dynamics</li> </ul> <p><b>DSE-7*:</b></p> <ul style="list-style-type: none"> <li>(i) Advanced Functional Analysis</li> <li>(ii) Algebraic Coding Theory</li> <li>(iii) Differential Geometry</li> <li>(iv) Finite Element Methods</li> </ul> <p>*Student will opt for DSE-7 or GE-3</p> | Developing Mathematical Ideas | <p><b>GE-3:</b></p> <ul style="list-style-type: none"> <li>(i) Dynamical Systems</li> <li>(ii) Numerical Methods for Ordinary Differential Equations</li> </ul> |

| Semester           | DSC  | DSE  | SBC                                       | GE  |
|--------------------|--|--|---|---|
| <b>Semester-IV</b> | <b>DSC-9:</b> Partial Differential Equations<br><b>DSC-10:</b> Analysis of Several Variables | <b>DSE-8:</b><br>(i) Advanced Fluid Dynamics<br>(ii) Probability Theory<br>(iii) Simplicial Homology Theory<br>(iv) Theory of Unbounded Operators<br><br><b>DSE-9:</b><br>(i) Banach and C*-Algebras<br>(ii) Chaos Theory<br>(iii) Character Theory of Finite Groups<br>(iv) Nonsmooth Optimization<br><br><b>DSE-10#:</b><br>(i) Computational Fluid Dynamics<br>(ii) Differential Topology<br>(iii) General Measure Theory<br>(iv) Theory of Non-commutative Rings<br><br>#Student will opt for DSE-10 or GE-4 | Workshops and Seminars on Advanced Topics | <b>GE-4:</b><br>(i) Nonsmooth Optimization<br>(ii) Probability Theory |

**Syllabi of Courses  
in  
Semester-III  
of  
M.Sc. Mathematics under  
Structure-1  
(Only Course work)**

## Discipline Specific Core (DSC) Courses

### DISCIPLINE SPECIFIC CORE – 7: FLUID DYNAMICS

#### CREDIT DISTRIBUTION OF THE COURSE

| Course Title & Code          | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any)                         |
|------------------------------|----------|-----------------------------------|----------|---------------------|---|--|
|                              |          | Lecture                           | Tutorial | Practical/ Practice |   |  |
| <b>DSC-7: Fluid Dynamics</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Calculus and Partial Differential Equations</b> |

#### Learning Objectives

The objective of this course is to:

- prepare a mathematical foundation to study the motion of fluids.
- develop concepts, models, and techniques to solve problems of fluid flow.
- develop the ability to conduct advanced studies and research in the broad field of fluid dynamics.

#### Learning Outcomes

After studying this course, the student will be able to:

- understand the concept of fluids, their classification, flow lines, models and approaches to study fluid flow.
- formulate mass and momentum conservation principles and obtain their solution for non-viscous flow.
- know potential flow, Bernoulli's equation, Kelvin's minimum energy and circulation theorems.
- understand two- and three-dimensional motion, complex potential, circle theorem, Blasius theorem, Weiss's and Butler's sphere theorems.
- apply the concept of stress and strain in viscous flow to derive Navier–Stokes equation of motion and energy equation.

#### Syllabus

##### Unit – 1

**(10 hours)**

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Flow lines, vorticity and circulation, Conservation of mass: Equation of continuity, Boundary surface.

##### Unit – 2

**(12 hours)**

Forces in fluid motion, Conservation of momentum: Euler's equation of motion, Theory of irrotational motion: Integration of Euler's equation under different conditions, Bernoulli's equation, Impulsive motion, Kelvin's minimum energy and circulation theorems, Potential theorem.

**Unit – 3**

**(13 hours)**

Two-dimensional motion: Complex potential, Line sources, sinks, doublets and vortices, Two-dimensional image system, Milne–Thomson circle theorem, Images with respect to a plane and cylinder, Blasius theorem. Three-dimensional flows, Weiss’s sphere theorem, Images with respect to sphere, Axi-symmetric flow, Stokes stream function, Butler’s sphere theorem, Flow past spheres and cylinders.

**Unit – 4**

**(10 hours)**

Stress and strain analysis, Newton’s law of viscosity, Laminar flow, Navier–Stokes equation of motion, Steady flow between parallel planes and Poiseuille flow, Constitutive equation, Energy equation.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.

[2] R. W. Fox, P. J. Pritchard and A. T. McDonald, *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.

[3] P. K. Kundu, I. M. Cohen and D. R. Dowling, *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

**Suggested Readings**

(i) L. M. Milne-Thomson, *Theoretical Hydrodynamics*, The Macmillan company, USA, 1969.

(ii) D. E. Rutherford, *Fluid Dynamics*, Oliver and Boyd Ltd., 1978.

**DISCIPLINE SPECIFIC CORE – 8: MEASURE AND INTEGRATION****CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                   | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any)                   |
|---------------------------------------|----------|-----------------------------------|----------|---------------------|---|--|
|                                       |          | Lecture                           | Tutorial | Practical/ Practice |   |  |
| <b>DSC-8: Measure and Integration</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Real Analysis and Riemann Integration</b> |

**Learning Objectives**

The primary objective of this course is to:

- extend the notion of length of an interval with the introduction of the concept of Lebesgue outer measure for any subset of real line.
- investigate the properties of Lebesgue measurable sets and functions.
- familiarize students with the Lebesgue integration of functions and its comparison with Riemann integration.
- generalize the concepts of measure and integration to an abstract space.

**Learning Outcomes**

This course will enable the students to:

- verify whether a given subset of  $\mathbb{R}$  or a real valued function is measurable.
- understand the requirement and the concept of the Lebesgue integral (a generalization of the Riemann integration) along with its properties.
- understand the statements and proofs of the fundamental integral convergence theorems and demonstrate their applications.
- carry out a comprehensive study of functions of bounded variation and their utility in understanding differentiation and integration.
- apply Hölder and Minkowski inequalities in  $L^p$ -spaces and understand completeness of  $L^p$ -spaces.

**Syllabus****Unit – 1****(14 hours)**

Lebesgue outer measure, Measurable sets, Lebesgue measure, Borel sets, Regularity, Measurable functions, Borel and Lebesgue measurability, Non-measurable sets.

**Unit – 2****(13 hours)**

Integration of nonnegative functions, General integral, Integration of series, Riemann and Lebesgue integrals.

**Unit – 3****(8 hours)**

Functions of bounded variation, Lebesgue's differentiation theorem, Differentiation and integration, Absolute continuity of functions.

**Unit – 4**

**(10 hours)**

Measures and outer measures, Measure spaces, Integration with respect to a measure,  $L^p$ -spaces, Hölder's and Minkowski's inequalities, Completeness of  $L^p$ -spaces.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

[1] G. de Barra, *Measure Theory and Integration*, Ellis Horwood Ltd., Chichester, John Wiley & Sons, Inc., New York, 1981 (Indian Reprint, 2014).

**Suggested Readings**

(i) M. Capinski and P. E. Kopp, *Measure, Integral and Probability*, Springer, 2005.

(ii) E. Hewitt and K. Stromberg, *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*, Springer, Berlin, 1975.

(iii) H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.

## Discipline Specific Elective (DSE) Courses

### DISCIPLINE SPECIFIC ELECTIVE – 5 (i): ALGEBRAIC TOPOLOGY

#### CREDIT DISTRIBUTION OF THE COURSE

| Course Title & Code                          | Credits  | Credit Distribution of the Course |          |                        | Eligibility Criteria                                  | Pre-requisite of the course (if any) |
|--|----------|-----------------------------------|----------|------------------------|---|--------------------------------------|
|  |          | Lecture                           | Tutorial | Practical/<br>Practice |   |                                      |
| <b>DSE-5 (i):<br/>Algebraic<br/>Topology</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>               | <b>Same as for entry<br/>to M.Sc.<br/>Mathematics</b> | <b>Basics of Topology</b>            |

#### Learning Objectives

The primary objective of this course is to:

- understand the concepts of homotopic maps, homotopy type, retracts, and deformation retracts, and their role in algebraic topology.
- compute fundamental groups of basic topological spaces.
- acquire knowledge of covering projections, lifting properties, Borsuk–Ulam theorem, and classification techniques of covering spaces.
- learn free groups and free products to understand and apply the Seifert–Van Kampen theorem.

#### Learning Outcomes

This course will enable the students to:

- distinguish between spaces with the same homotopy type.
- compute fundamental groups of standard spaces such as  $n$ -sphere  $S^n$  and punctured planes.
- apply the concept of fundamental groups to prove Brouwer’s fixed-point theorem and the Fundamental theorem of Algebra.
- explain the lifting theorems and their implications in topology.
- classify covering spaces for given base spaces.
- apply Seifert–Van Kampen theorem to compute fundamental groups of glued spaces.

#### Syllabus

##### **Unit – 1** **(11 hours)**

Homotopic maps, Homotopy type, Retract and deformation retract.

##### **Unit – 2** **(12 hours)**

Fundamental group, Calculation of fundamental groups of  $n$ -sphere  $S^n$  and punctured plane, Brouwer’s fixed-point theorem, Fundamental theorem of Algebra.

##### **Unit – 3** **(12 hours)**

Covering projections, Lifting theorems, Borsuk–Ulam theorem, Classification of covering spaces.

##### **Unit – 4** **(10 hours)**

Free products, Free groups, Seifert–Van Kampen theorem and its applications.

### Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

### Essential Readings

- [1] J. R. Munkres, *Elements of Algebraic Topology*, Addison-Wesley Publishing Company, 1984.
- [2] T. B. Singh, *Introduction to Topology*, Springer Nature Singapore, 2019.

### Suggested Readings

- (i) G. E. Bredon, *Geometry and Topology*, Springer, 2014.
- (ii) A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
- (iii) W. S. Massey, *A Basic Course in Algebraic Topology*, World Publishing Corporation, 2009.
- (iv) J. J. Rotman, *An Introduction to Algebraic Topology*, Springer, 2011.
- (v) E. H. Spanier, *Algebraic Topology*, Springer-Verlag, 1989.

**DISCIPLINE SPECIFIC ELECTIVE – 5 (ii): COMMUTATIVE ALGEBRA****CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                    | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any)             |
|--|----------|-----------------------------------|----------|---------------------|---|--|
|  |          | Lecture                           | Tutorial | Practical/ Practice |   |  |
| <b>DSE-5 (ii): Commutative Algebra</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Linear Algebra and Field Theory</b> |

**Learning Objectives**

The objective of this course is to:

- develop a solid understanding of the structure of commutative rings, ideals, their radicals, extension, contraction etc.
- study important constructions such as total quotient rings, localizations.
- develop basic foundation in other areas of mathematics such as algebraic geometry, algebraic number theory.

**Learning Outcomes**

This course will enable the students to:

- know the localization of rings at a prime ideal that is an algebraic analogue of the geometric notion concentrating attention near a point.
- know more closely the polynomial rings, power series rings in one or more variables over a commutative ring and their prime spectrum.
- define, identify, and elaborate integral closure of rings, valuations rings, discrete valuation rings, structure theorem of Artin rings.

**Syllabus****Unit – 1 (12 hours)**

Radical of an ideal, Prime avoidance lemma, Chinese remainder theorem, Extension and contraction of ideals, Jacobson radical of a ring, Nakayama lemma, Tensor product of modules.

**Unit – 2 (13 hours)**

Rings and modules of fractions, Localization, Local properties, Primary decomposition, First and second uniqueness theorem of primary decomposition, Associated prime ideals of decomposable ideals.

**Unit – 3 (10 hours)**

Integral ring extensions, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem.

**Unit – 4 (10 hours)**

Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings, Discrete valuation rings.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

[1] M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

**Suggested Readings**

- (i) D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 2004.
- (ii) R. Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press, 2000.
- (iii) B. Singh, *Basic Commutative Algebra*, World Scientific, 2011.
- (iv) O. Zariski and P. Samuel, *Commutative Algebra*, Volume I & II, Springer, 1975.

## DISCIPLINE SPECIFIC ELECTIVE – 5 (iii): DYNAMICAL SYSTEMS

### CREDIT DISTRIBUTION OF THE COURSE

| Course Title & Code                       | Credits  | Credit Distribution of the Course |          |                        | Eligibility Criteria                          | Pre-requisite of the course (if any)                          |
|---|----------|-----------------------------------|----------|------------------------|---|---|
|   |          | Lecture                           | Tutorial | Practical/<br>Practice |   |   |
| <b>DSE-5 (iii):<br/>Dynamical Systems</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>               | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Topology and Ordinary Differential Equations</b> |

### Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

### Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

### Syllabus

#### **Unit – 1 (13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

#### **Unit – 2 (10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

#### **Unit – 3 (12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincaré-Bendixson theorem.

#### **Unit – 4 (10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

### Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

- [1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
- [2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

**Suggested Readings**

- (i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
- (ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
- (iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

**DISCIPLINE SPECIFIC ELECTIVE – 5 (iv): THEORY OF BOUNDED OPERATORS****CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                            | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any) |
|--|----------|-----------------------------------|----------|---------------------|---|--------------------------------------|
|  |          | Lecture                           | Tutorial | Practical/ Practice |   |                                      |
| <b>DSE-5 (iv): Theory of Bounded Operators</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Functional Analysis</b> |

**Learning Objectives**

The primary objective of this course is to:

- introduce some classes of bounded linear operators which play a central role in both pure and applied mathematics.
- study the properties and spectral theory of these operators.

**Learning Outcomes**

This course will enable the students to understand:

- the spectrum and sub-divisions of spectrum of standard operators like shifts and multiplication.
- the spectral theorem for some classes of bounded linear operators.
- the concepts of compactness, self-adjointness and positivity of bounded linear operators.
- trace class and Hilbert–Schmidt operators.

**Syllabus****Unit – 1****(11 hours)**

Properties of spectrum and resolvent of bounded operators, Subdivision of the spectrum including point, approximate and compression spectrum.

**Unit – 2****(10 hours)**

Operators on Hilbert spaces, Adjoint operator, Projections and idempotents, Operations with projections, Invariant and reducing subspaces.

**Unit – 3****(14 hours)**

Compact operators on Hilbert spaces, Diagonalisation of compact self-adjoint operators, Spectral theorem and functional calculus for Compact normal operators, Positive operators, Compact operators on Banach spaces, Spectral theory of compact operators.

**Unit – 4****(10 hours)**

Polar decomposition, Singular values, Trace class operators, Trace norm and Hilbert Schmidt operators.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

### Essential Readings

[1] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2009.

[2] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.

### Suggested Readings

(i) E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2006.

(ii) B. Simon, *Operator Theory: A Comprehensive Course in Analysis*, Part 4, American Mathematical Society, 2015.

(iii) S. R. Garcia, J. Mashregi and W. T. Ross, *Operator Theory by Example*, Oxford University Press, 2023.

**DISCIPLINE SPECIFIC ELECTIVE – 6 (i): ADVANCED COMPLEX ANALYSIS****CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                                     | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any) |
|---|----------|-----------------------------------|----------|---------------------|---|--------------------------------------|
|   |          | Lecture                           | Tutorial | Practical/ Practice |   |                                      |
| <b>DSE-6 (i):<br/>Advanced<br/>Complex<br/>Analysis</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Complex Analysis</b>    |

**Learning Objectives**

The primary objective of this course is to:

- explore the metric space structure of the spaces of continuous and analytic functions, through Arzela–Ascoli, Hurwitz and Montel theorems.
- investigate various characterizations of simply connected regions, with a special focus on the Riemann mapping theorem.
- use Runge’s and Mittag–Leffler’s theorems to approximate and interpolate analytic and meromorphic functions.
- analyze the range of analytic functions using Bloch’s/ Landau’s constants and Picard’s theorem.

**Learning Outcomes**

This course will enable the students to:

- apply various variants of maximum modulus theorem, encompassing Hadamard’s three circles theorem and Phragmen–Lindelöf theorem.
- comprehend the notions of normality, compactness, equicontinuity and local boundedness for the spaces of continuous and analytic functions.
- construct and factorize entire functions using infinite products, including special functions like gamma and zeta.
- analyze the harmonic functions on a disk using Poisson kernel, which in turn, solves the Dirichlet problem for a unit disk.

**Syllabus****Unit – 1****(12 hours)**

Convex functions and Hadamard’s three circles theorem, Maximum-Modulus theorem (third version), Phragmen–Lindelöf theorem, Spaces of continuous functions, Normal and equicontinuous families, Arzela–Ascoli theorem.

**Unit – 2****(11 hours)**

Spaces of analytic functions, Hurwitz’s theorem, Montel’s theorem, Riemann mapping theorem, Infinite products, Weierstrass factorization theorem, Factorization of sine function.

**Unit – 3****(10 hours)**

Gamma and Riemann zeta function, Runge’s theorem, Characterizations of simple connectedness, Mittag–Leffler’s theorem.

**Unit – 4**

**(12 hours)**

Harmonic functions, Mean value property, Maximum and minimum principles, Harmonic function on a disk, Harnack's theorem, Range of an analytic function, Bloch's theorem, Bloch's and Landau's constants, Picard's theorem.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

[1] J. B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa Publishing House, New Delhi, 2002.

**Suggested Readings**

- (i) L. V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
- (ii) L. Hahn and B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
- (iii) E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press, 2003.
- (iv) D. Ullrich, *Complex Made Simple*, Volume 97, American Mathematical Society, 2008.

**DISCIPLINE SPECIFIC ELECTIVE – 6 (ii): NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS**
**CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code  | Credits  | Credit Distribution of the Course |          |                        | Eligibility Criteria                                  | Pre-requisite of the course (if any)                     |
|--|----------|-----------------------------------|----------|------------------------|---|--|
|  |          | Lecture                           | Tutorial | Practical/<br>Practice |   |  |
| <b>DSE-6 (ii):<br/>Numerical Methods<br/>for Ordinary<br/>Differential<br/>Equations</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>               | <b>Same as for entry<br/>to M.Sc.<br/>Mathematics</b> | <b>Basics of Ordinary<br/>Differential<br/>Equations</b> |

**Learning Objectives**

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

**Learning Outcomes**

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

**Syllabus**
**Unit – 1**
**(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

**Unit – 2**
**(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

**Unit – 3**
**(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

#### Unit – 4

(10 hours)

Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

#### Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

#### Essential Readings

- [1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.
- [2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

#### Suggested Readings

- (i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.
- (ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.
- (iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.

**DISCIPLINE SPECIFIC ELECTIVE – 6 (iii): REPRESENTATION OF FINITE GROUPS**
**CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                                 | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any)             |
|---|----------|-----------------------------------|----------|---------------------|---|--|
|   |          | Lecture                           | Tutorial | Practical/ Practice |   |  |
| <b>DSE-6 (iii): Representation of Finite Groups</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Linear Algebra and Group Theory</b> |

**Learning Objectives**

The primary objective of this course is to:

- represent finite groups as groups of matrices (via homomorphisms) and apply the tools of linear algebra to study the group structure.
- introduce the notion of Group algebra, which plays an essential role in classifying representations of groups.
- to discuss some applications of representations of finite groups, such as the Burnside's theorem.

**Learning Outcomes**

This course will enable the students to:

- define and construct examples of group representations,  $FG$ -modules, group algebras.
- grasp key concepts and tools of representation theory and establish a link between  $FG$ -modules and group representations.
- prove and apply Maschke's theorem and Schur's lemma to describe all irreducible representations of finite groups over the field of complex numbers.
- apply the theory of characters and group representations to gain insight into group structure, such as normal subgroups, and the solubility of groups.

**Syllabus**
**Unit – 1**
**(11 hours)**

Representation of groups,  $FG$ -modules and  $FG$ -submodules, and reducibility, Permutation modules,  $FG$ -modules and equivalent representations, Reducible and irreducible  $FG$ -modules, Group algebra of  $G$ , Regular  $FG$ -module and regular representations,  $FG$ -homomorphisms, Direct sum of  $FG$ -modules.

**Unit – 2**
**(11 hours)**

Maschke's theorem for  $FG$ -modules and consequences. Schur's lemma and its converse, Application of Schur's lemma, Irreducible modules and group algebra, Structure of group algebra and space of  $CG$ -homomorphisms.

**Unit – 3**
**(10 hours)**

Characters and their properties, Permutation and regular characters, Inner product, Number of irreducible characters, Orthogonality relations and finding normal subgroups.

**Unit – 4**

**(13 hours)**

Algebraic numbers, Algebraic integers and their properties, Character values, The Burnside's  $(p,q)$ -theorem and solubility of some particular groups.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

[1] G. James and M. Liebeck, *Representations and Characters of Groups*, Second Edition, Cambridge University Press, 2005.

**Suggested Readings**

- (i) C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, American Mathematical Society, 2006.
- (ii) W. Fulton and J. Harris, *Representation Theory - A First Course*, Springer-Verlag, 2004.
- (iii) I. M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.

**DISCIPLINE SPECIFIC ELECTIVE – 6 (iv): TOPOLOGICAL DYNAMICS****CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                     | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any) |
|---|----------|-----------------------------------|----------|---------------------|---|--------------------------------------|
|   |          | Lecture                           | Tutorial | Practical/ Practice |   |                                      |
| <b>DSE-6 (iv): Topological Dynamics</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Topology</b>            |

**Learning Objectives**

The primary objective of this course is to:

- provide a strong background of topological dynamical systems including their applications.
- develop some useful and interesting dynamical properties like expansivity, shadowing and topological stability with supporting examples and results from symbolic and topological dynamics.
- introduce the celebrated Sarkovskii's theorem.

**Learning Outcomes**

This course will enable the students to:

- construct interesting examples of dynamical systems and topological conjugacy.
- visualize stable sets, omega sets and alpha limit sets.
- understand the applications of Sarkovskii's theorem.
- use subshifts of finite type to characterize irreducible matrices.
- prove key results on expansivity and shadowing regarding existence/non-existence, product, subspace and their different characterizations etc.
- find the class of topologically stable homeomorphisms.

**Syllabus****Unit – 1****(10 hours)**

Definition and examples (including real life examples) of dynamical systems, Orbits, Types of orbits, Topological conjugacy and orbits, Phase portrait-graphical analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties.

**Unit – 2****(10 hours)**

Sarkovskii's theorem, Shift spaces and subshift, Subshift of finite type, Subshift represented by a matrix, Characterizations of irreducible matrices.

**Unit – 3****(13 hours)**

Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms.

**Unit – 4****(12 hours)**

Converging semi-orbits for expansive homeomorphisms, Definition, examples and properties of maps having shadowing property, Topological Anosov homeomorphisms and topological stability.

### Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

### Essential Readings

- [1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.
- [2] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2004.

### Suggested Readings

- (i) D. C. Hanselman and B. Little field, *Mastering MATLAB*, Pearson, 2012.
- (ii) D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1996.
- (iii) C. Robinson, *Dynamical Systems, Stability, Symbolic Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 1998.
- (iv) J. de. Vries, *Elements of Topological Dynamics*, Springer, 1993.

**DISCIPLINE SPECIFIC ELECTIVE – 7 (i): ADVANCED FUNCTIONAL ANALYSIS**
**CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                            | Credits  | Credit Distribution of the Course |          |                        | Eligibility Criteria                          | Pre-requisite of the course (if any)              |
|--|----------|-----------------------------------|----------|------------------------|---|---|
|  |          | Lecture                           | Tutorial | Practical/<br>Practice |   |   |
| <b>DSE-7 (i): Advanced Functional Analysis</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>               | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Functional Analysis and Topology</b> |

### Learning Objectives

The primary objective of this course is to:

- define and explain the structure of a topological vector space and its fundamental properties.
- differentiate between normed, metrizable, locally convex and Hausdorff topological vector spaces.
- introduce the foundational theorems of functional analysis, including the Hahn–Banach, Banach–Steinhaus, Open mapping, and Closed graph theorems in the context of locally convex spaces.
- explain some applications of Banach–Alaoglu theorem and Krein–Milman theorem.

### Learning Outcomes

This course will enable the students to:

- appreciate types of topological vector spaces and their separation properties.
- understand quotient spaces, weak topology and weak\*-topology.
- analyze concepts of continuity, boundedness, and convergence for linear operators and functionals on topological vector spaces.
- understand the notion of local convexity and the role of seminorms in defining locally convex topologies.

### Syllabus

#### Unit – 1

**(12 hours)**

Topological vector spaces, Types of Topological vector spaces, Separation properties, Linear mappings, Finite dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Normability.

#### Unit – 2

**(11 hours)**

Quotient spaces, Seminorms and quotient spaces, Examples, Baire category theorem, Banach–Steinhaus theorem, The open mapping theorem and the closed graph theorem on topological vector spaces.

#### Unit – 3

**(11 hours)**

Hahn–Banach separation theorem on topological vector spaces, Continuous extension theorem, Weak topologies, Weak topology and convexity, Weak topology and metrizability, Weak\*-topology of a dual space, Compact convex sets, Banach–Alaoglu theorem and applications, Goldstine theorem.

**Unit – 4**

**(11 hours)**

Extreme points, Krein–Milman theorem, Convex hull of compact sets, Applications of Krein–Milman theorem: Stone–Weierstrass theorem, Markov–Kakutani fixed point theorem.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

[1] J. B. Conway, *A Course in Functional Analysis*, Second Edition, Springer, 2007.

[2] W. Rudin, *Functional Analysis*, Second Edition, Tata Mc Graw-Hill, 2011.

**Suggested Readings**

(i) V. I. Bogachev and O. G. Smolyanov, *Topological Vector Spaces and Their Applications*, Springer, 2017.

(ii) S. Kantorovitz, *Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics, Oxford University Press, 2006.

(iii) J. Voigt, *A Course on Topological Vector Spaces*, Birkhäuser, 2020.

**DISCIPLINE SPECIFIC ELECTIVE – 7 (ii): ALGEBRAIC CODING THEORY****CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                        | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any)              |
|--|----------|-----------------------------------|----------|---------------------|---|---|
|  |          | Lecture                           | Tutorial | Practical/ Practice |   |   |
| <b>DSE-7 (ii): Algebraic Coding Theory</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Linear Algebra, Groups and Rings</b> |

**Learning Objectives**

The primary objective of this course is to:

- provide an introduction to algebraic coding theory, particularly linear codes.
- discuss bounds on the parameters along with cyclic codes.
- describe some well-known codes, such as Reed–Muller and Golay codes.
- explore the algebraic structure of Cyclic and Quadratic residue codes over fields and rings.

**Learning Outcomes**

This course will enable the students to:

- get an insight into the matrix representation of a code, as well as encoding and decoding.
- understand Hamming, MDS and Reed–Muller codes.
- describe cyclic codes and their generator polynomial.
- learn about special cyclic codes, such as Quadratic residue codes, and their properties over the ring  $\mathbb{Z}_4$ .

**Syllabus****Unit – 1****(10 hours)**

Error detecting and error correcting codes, Maximum likelihood decoding, Hamming distance, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Encoding and decoding of linear codes, Syndrome decoding.

**Unit – 2****(11 hours)**

Bounds on codes, Sphere covering bound, Hamming bound, Perfect codes, Binary Hamming codes, Binary Golay codes, Singleton bound and MDS codes. Propagation rules, Reed–Muller codes.

**Unit – 3****(12 hours)**

Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Generator and parity-check matrices of cyclic codes, Decoding of cyclic codes, Burst error correcting codes.

**Unit – 4****(12 hours)**

Quadratic residue codes: QR codes over fields of characteristic 2 and 3, Cyclic codes and their generating polynomial over  $\mathbb{Z}_4$ , QR codes over  $\mathbb{Z}_4$ .

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials

along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

- [1] S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004.
- [2] W. C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2010.

**Suggested Readings**

- (i) R. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.
- (ii) F. J. Mac William and N. J. A. Sloane, *Theory of Error Correcting Codes, Part I & II*, Elsevier/North-Holland, Amsterdam, 1977.

**DISCIPLINE SPECIFIC ELECTIVE – 7 (iii): DIFFERENTIAL GEOMETRY****CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                       | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any)  |
|---|----------|-----------------------------------|----------|---------------------|---|---|
|   |          | Lecture                           | Tutorial | Practical/ Practice |   |   |
| <b>DSE-7 (iii): Differential Geometry</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Linear Algebra, Multivariate Calculus and Differential Equations</b> |

**Learning Objectives**

The primary objective of this course is to introduce:

- surfaces and parametrized surfaces.
- orientation on connected surfaces.
- geodesics on surfaces.
- Weingarten maps on oriented surfaces.
- arc length and curvature of oriented plane curves.
- curvatures of oriented surfaces.

**Learning Outcomes**

This course will enable the students to:

- understand the concepts of level sets and graphs of functions, smooth vector fields, tangent spaces of level sets.
- appreciate surfaces and parametrized surfaces, Gauss map, geodesics and parallel transport on oriented surfaces.
- know what the Weingarten map of an oriented surface is, realize it as shape operator and use it to compute curvature of oriented plane curves.
- find global parametrization and hence arc length of an oriented plane curve.
- compute various types of curvatures of surfaces.

**Syllabus****Unit – 1****(10 hours)**

Level sets in  $\mathbb{R}^{n+1}$  and graphs of functions, Smooth vector fields and existence and uniqueness of their integral curves, Tangent spaces of level sets at regular points, Surfaces in  $\mathbb{R}^{n+1}$  as inverse images of regular values of smooth functions, Necessary condition for extrema of functions on surfaces-Lagrange multipliers, Existence of a normal vector field on a connected surface, Orientation, Gauss map.

**Unit – 2****(13 hours)**

The notion of a geodesic on a surface, Existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof, Covariant derivative of a smooth vector field, Parallel vector field along a curve, Existence and uniqueness of a parallel vector field along a curve with a given initial vector, Weingarten map of a surface at a point, Local parametrization and curvature of a plane curve.

**Unit – 3**

**(10 hours)**

Global parametrization and arc length of an oriented plane curve, Differential 1-forms, Line integral of 1-forms over parametrized curves.

**Unit – 4**

**(12 hours)**

Parametrized surfaces with examples, Curvature of surfaces, Normal curvature of a surface at a point in a given direction, Principal curvatures, First and second fundamental forms, Gauss-Kronecker curvature and mean curvature.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

[1] A. Pressley, *Elementary Differential Geometry*, Springer-Verlag London Limited, 2012.

[2] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer (India) Pvt. Limited, 2004.

**Suggested Readings**

(i) W. Kuhnel, *Differential Geometry: Curves-Surfaces-Manifolds*, Third Edition, American Mathematical Society, 2015.

(ii) B. O' Neill, *Elementary Differential Geometry*, Second Edition, Academic Press INC., Academic Press, New York, 2006.

**DISCIPLINE SPECIFIC ELECTIVE – 7 (iv): FINITE ELEMENT METHODS****CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code                           | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any)    |
|---|----------|-----------------------------------|----------|---------------------|---|---|
|   |          | Lecture                           | Tutorial | Practical/ Practice |   |   |
| <b>DSE-7 (iv):<br/>Finite Element Methods</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>Basics of Differential Equations</b> |

**Learning Objectives**

The primary objective of this course is to:

- introduce basic aspects of finite element methods (FEM) including domain discretization, polynomial interpolation, application of boundary conditions, assembly of global arrays, and solution of the resulting algebraic systems.
- discuss the use of finite element methods in solving engineering problems in the domain of solid mechanics, fluid mechanics, heat transfer and electromagnetism.

**Learning Outcomes**

This course will enable the students to:

- use integral statement to deduce finite element approximations for the underlying linear partial differential equations.
- write special-purpose finite element programs within a procedural programming environment.
- use finite element methods to solve engineering problems in solids mechanics, fluid mechanics, heat transfer, and electromagnetism.
- assess the accuracy and reliability of finite element solutions and troubleshoot problems arising from errors in a given finite element analysis.

**Syllabus****Unit – 1****(12 hours)**

Basic concepts of weak formulation, Variational formulation of a one dimensional model equation, Basis function and finite element solutions, Collocation method, Ritz method, Least square method, Standard Galerkin method, FEM for model problem, Error estimate for FEM for model equation, Convergence analysis.

**Unit – 2****(11 hours)**

Various shapes of finite element, Higher order basis functions, Finite element methods for elliptic problems: Variational methods, Standard Galerkin method, Error estimate for FEM for elliptic problem, FEM for Poisson equation.

**Unit – 3****(12 hours)**

Finite element methods for parabolic problems: One dimensional model problems, Semi-discretization in space, Error estimates, Discretization in space and time, Galerkin method, Finite element methods for hyperbolic problems: Standard Galerkin method, Standard Galerkin method with strongly and weakly imposed boundary conditions.

**Unit – 4**

**(10 hours)**

Applications of the FEM to second order BVPs in one dimension, Applications of the FEM to linear elliptic, parabolic and hyperbolic equations.

**Tutorial**

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

**Essential Readings**

[1] G. Evans, J. Blackledge and P. Yardley, *Numerical Methods for Partial Differential Equations*, Springer-Verlag, London, 2000.

[2] C. Johnson, *Numerical Solutions of Partial Differential Equations by Finite Element Methods*, Cambridge University Press, Cambridge, 1987.

[3] J. Whiteley, *Finite Element Methods - A Practical Guide*, Springer, 2016.

**Suggested Readings**

(i) Z. Chen, *Finite Element Methods and Their Applications*, Springer-Verlag, New York, 2005.

(ii) V. Thomee, *Galerkin Finite Element Methods for Parabolic Problems*, Second Edition, Springer-Verlag, Berlin, 2006.

## Skill-Based Course (SBC)

### DEVELOPING MATHEMATICAL IDEAS

| Course Title                         | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                          | Pre-requisite of the course (if any) |
|--------------------------------------|----------|-----------------------------------|----------|---------------------|---|--------------------------------------|
|                                      |          | Lecture                           | Tutorial | Practical/ Practice |   |                                      |
| <b>Developing Mathematical Ideas</b> | <b>2</b> | <b>0</b>                          | <b>0</b> | <b>2</b>            | <b>Same as for entry to M.Sc. Mathematics</b> | <b>NIL</b>                           |

#### Learning Objectives

This course will train students

- to develop skills to create new mathematical ideas independently.
- to present these ideas adeptly.

#### Learning Outcomes

Students will be able to

- hone their analytical skills and their ability to think critically.
- learn to work collaboratively.
- acquire skills that help them to create and develop new mathematical ideas.

#### Methodology

We plan to form groups of students, say of 5-6 each, who will be assigned a piece of mathematical work (article/ paper published in reputed journals/ periodicals/book chapters). The designated groups will be required to read and understand this mathematical work under the supervision of a faculty member. They will be further encouraged to pose meaningful questions/problems in the context of the mathematics they have read and possibly offer solutions. Presentations will be conducted for these groups as a part of their assessment process.

Appropriate material for the study will be provided by the department/faculty. Papers/articles for example, may be chosen from resources like *Involve* (Link: <https://msp.org/involve/>), *SIAM Undergraduate Research Online* (Link: <https://www.siam.org/publications/siuro>), *The American Mathematical Monthly* (Mathematical Association of America, Taylor and Francis, Link: <https://www.tandfonline.com/journals/uamm20>), *Mathematics Magazine* (Mathematical Association of America, Taylor and Francis, Link: <https://www.tandfonline.com/journals/umma20>), *The Mathematics Student* (Indian Mathematical Society, Link: <https://www.indianmathsoc.org/MS.html>) and *The Mathematical Intelligencer* (Springer Nature, Link: <https://link.springer.com/journal/283>). Advanced topics beyond the prescribed syllabus from textbooks/ research monographs may also be chosen.

## Generic Elective (GE) Courses

### GENERIC ELECTIVE – 3 (i): DYNAMICAL SYSTEMS

#### CREDIT DISTRIBUTION OF THE COURSE

| Course Title & Code                    | Credits  | Credit Distribution of the Course |          |                        | Eligibility Criteria                   | Pre-requisite of the course (if any)                          |
|--|----------|-----------------------------------|----------|------------------------|--|---|
|  |          | Lecture                           | Tutorial | Practical/<br>Practice |  |   |
| <b>GE-3 (i):<br/>Dynamical Systems</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>               | <b>Class XII pass with Mathematics</b> | <b>Basics of Topology and Ordinary Differential Equations</b> |

#### Learning Objectives

The primary objective of this course is to:

- understand discrete and continuous systems with case studies to study nonlinear systems of ordinary differential equations and dynamical systems.
- understand the concepts, models and techniques to realize the real-world problems and stability of the systems along with the chaotic dynamic behaviour of models by understanding bifurcations.

#### Learning Outcomes

This course will enable the students to learn:

- formulation of mathematical models with the stability analysis near the equilibrium points.
- how the concept of phase portraits helps to analyse mathematical model graphically.
- the qualitative behaviour of the solution set of a given system of differential equations including the invariant sets and limiting behaviour of the dynamical system or flow defined by the system of differential equations.
- how different bifurcations explain the chaotic behaviour of the system.

#### Syllabus

##### **Unit – 1 (13 hours)**

Linear systems: Jordan forms, Stability theory; Nonlinear systems: Fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, Flow of a differential equation, Linearization, Stable manifold theorem, Hartman–Grobman theorem.

##### **Unit – 2 (10 hours)**

Stability and Lyapunov functions, Saddle points, Nodes, Foci, Centers and nonhyperbolic critical points, Center manifold theorem.

##### **Unit – 3 (12 hours)**

Limit sets and attractors, Periodic orbits and limit cycles, Poincaré map, Stable manifold theorem for periodic orbits, Poincaré–Bendixson theorem.

##### **Unit – 4 (10 hours)**

Bifurcations at nonhyperbolic equilibrium points, Saddle node, Transcritical and Pitchfork bifurcations, Hopf bifurcation.

### Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

### Essential Readings

- [1] M. W. Hirsch, S. Smale and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.
- [2] L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer Verlag, 2001.

### Suggested Readings

- (i) R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.
- (ii) S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.
- (iii) S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, TAM Volume 2, Springer-Verlag, NY, 1990.

**GENERIC ELECTIVE – 3 (ii): NUMERICAL METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS**
**CREDIT DISTRIBUTION OF THE COURSE**

| Course Title & Code   | Credits  | Credit Distribution of the Course |          |                     | Eligibility Criteria                   | Pre-requisite of the course (if any)             |
|---|----------|-----------------------------------|----------|---------------------|--|--|
|   |          | Lecture                           | Tutorial | Practical/ Practice |  |  |
| <b>GE-3 (ii): Numerical Methods for Ordinary Differential Equations</b> | <b>4</b> | <b>3</b>                          | <b>1</b> | <b>0</b>            | <b>Class XII pass with Mathematics</b> | <b>Basics of Ordinary Differential Equations</b> |

**Learning Objectives**

The primary objective of this course is to:

- develop the basic theory underlying the numerical solution of differential equations.
- introduce the concepts of consistency, stability and convergence of finite difference methods.
- execute the numerical schemes for the solution of differential equations.

**Learning Outcomes**

This course will enable the students to:

- gain a thorough understanding of the fundamental concepts involved in the construction and analysis of finite difference schemes for solving ordinary differential equations (ODEs).
- apply various numerical methods based on finite difference approaches to obtain approximate solutions for both initial value problems (IVPs) and boundary value problems (BVPs).
- develop the ability to select appropriate finite difference methods for specific types of problems and effectively apply them to real world applications.

**Syllabus**
**Unit – 1**
**(11 hours)**

Initial value problems: Existence and uniqueness of solution, Finite difference equation, Truncation error, Single step methods for first order IVPs and system of IVPs- Family of explicit and implicit Runge–Kutta methods, Taylor series methods, Derivation, Truncation error, Consistency, Stability and convergence analysis.

**Unit – 2**
**(12 hours)**

IVPs for the system of ODEs, Consistency, Zero stability and convergence of linear multistep methods, Routh–Hurwitz criterion, Order and error constant, First Dahlquist Barrier, Local truncation error and global truncation error, Error bounds, Local error, Linear stability theory, Higher order differential equations.

**Unit – 3**
**(12 hours)**

Derivation of explicit and implicit multistep methods for IVPs and system of IVPs, Truncation error, Stability and convergence analysis of family of Nystrom method, Adams–Bashforth method,

Adams–Moulton method, Milne–Simpson method, Predictor corrector method, and Modified predictor corrector method, Hybrid method, Multistep methods for second order IVPs.

#### Unit – 4

(10 hours)

Linear BVPs for second order ordinary differential equations, Shooting method, Finite difference method, Collocation method, Derivative boundary conditions, Nonlinear two-point BVPs, Higher order finite difference methods, Stability, Truncation error and convergence analysis.

#### Tutorial

Problem-solving sessions based on material covered in the lectures will be taken up in the tutorials along with scholastic work related to conceptual understanding of the subject.

#### Essential Readings

- [1] M. K. Jain, S. R. K. Iyenger and R. K. Jain, *Numerical Methods for Scientific and Engineering Computations*, Seventh Edition, New Age International Publisher, 2019.
- [2] J. D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

#### Suggested Readings

- (i) K. E. Atkinson, W. Han and D. E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.
- (ii) J. C. Butcher, *The Numerical Analysis of Ordinary Differential Equations*, Second Edition, Wiley, New York, 2008.
- (iii) L. Collatz, *The Numerical Treatment of Differential Equations*, Springer-Verlag, 1966.